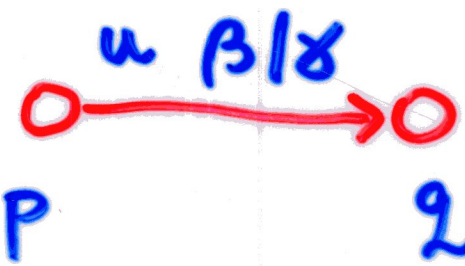
STATE DIAGRAMS FOR PUSH DOWN AUTOMATA



means

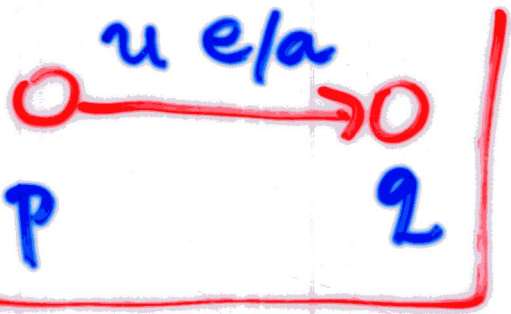
$$((p, u, \beta)(q, \gamma)) \in \Delta$$

i.e

In State (p)

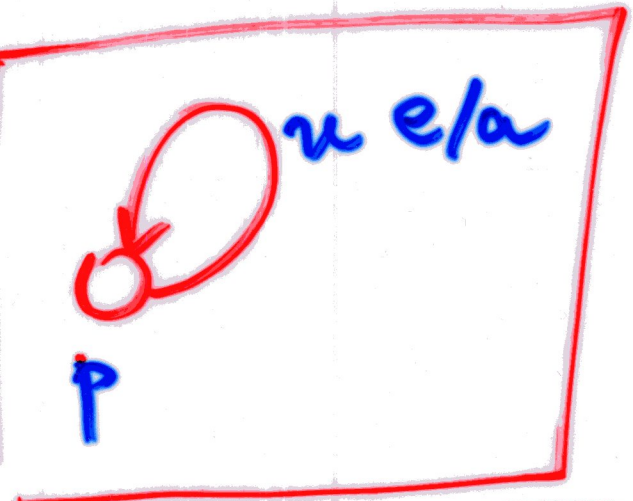
1. read (u)
2. replace (β) by (γ) on the top of the stack
3. go to the state (q)

PUSH a on the stack while reading u



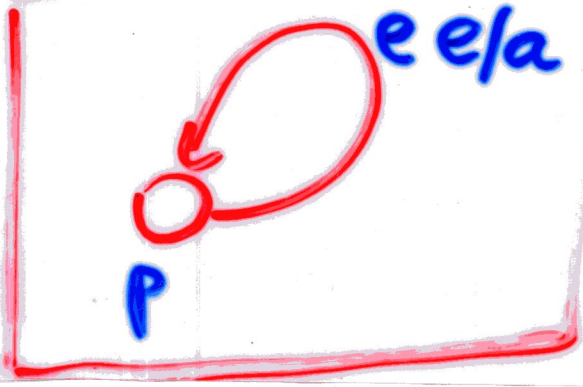
$\equiv$ $((p, u, e), (q, a)) \in \Delta$

↑ This pushes a with transition from state p to q



push a with No change of state (still reading u)

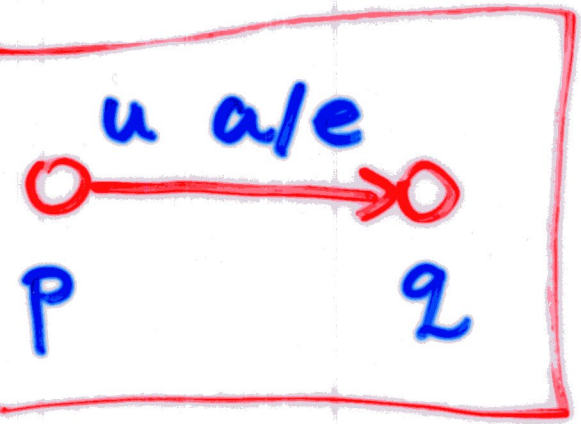
$((p, u, e)(p, a)) \in \Delta$



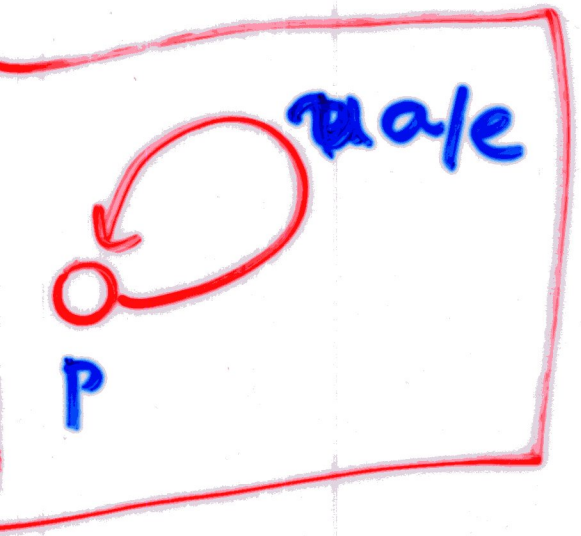
PUSH a no change of state, reading nothing

$((p, e, e)(p, a)) \in \Delta$

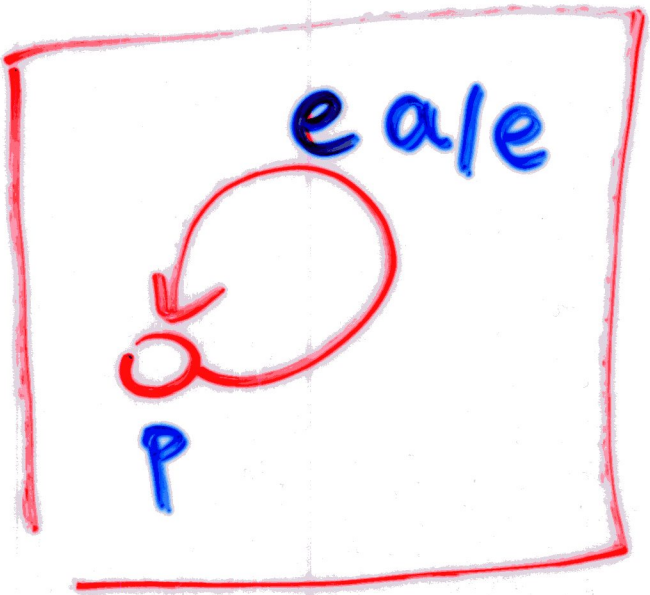
POP a



In state (p) read u REMOVE a from top of stack go to (q)

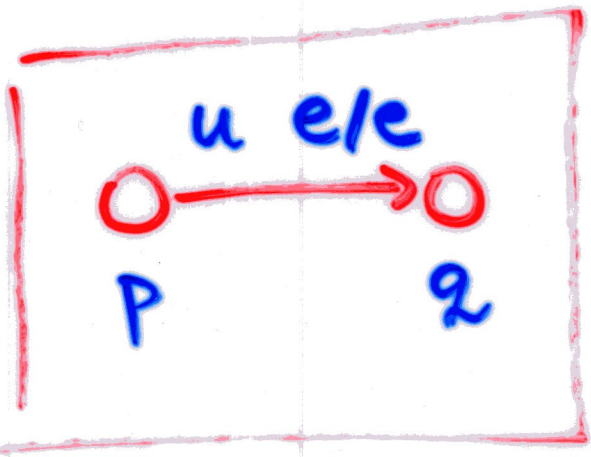


pop a with no change of state reading u



In state p POP a reading nothing no change of state

Push Down M' " " "



imitates

finite automata

$\Gamma = \phi$ stack alph

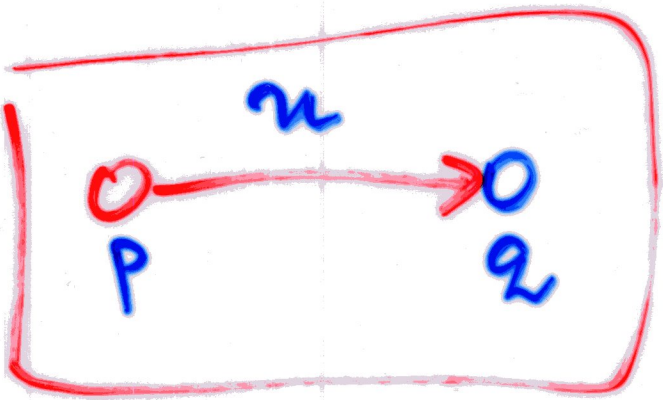
$(p, u, q) \in \Delta$ → finite M if f

$e \in \Gamma^* = \phi^*$

$((p, u, e)(q, e)) \in \Delta'$

↓

push down M'



Finite M

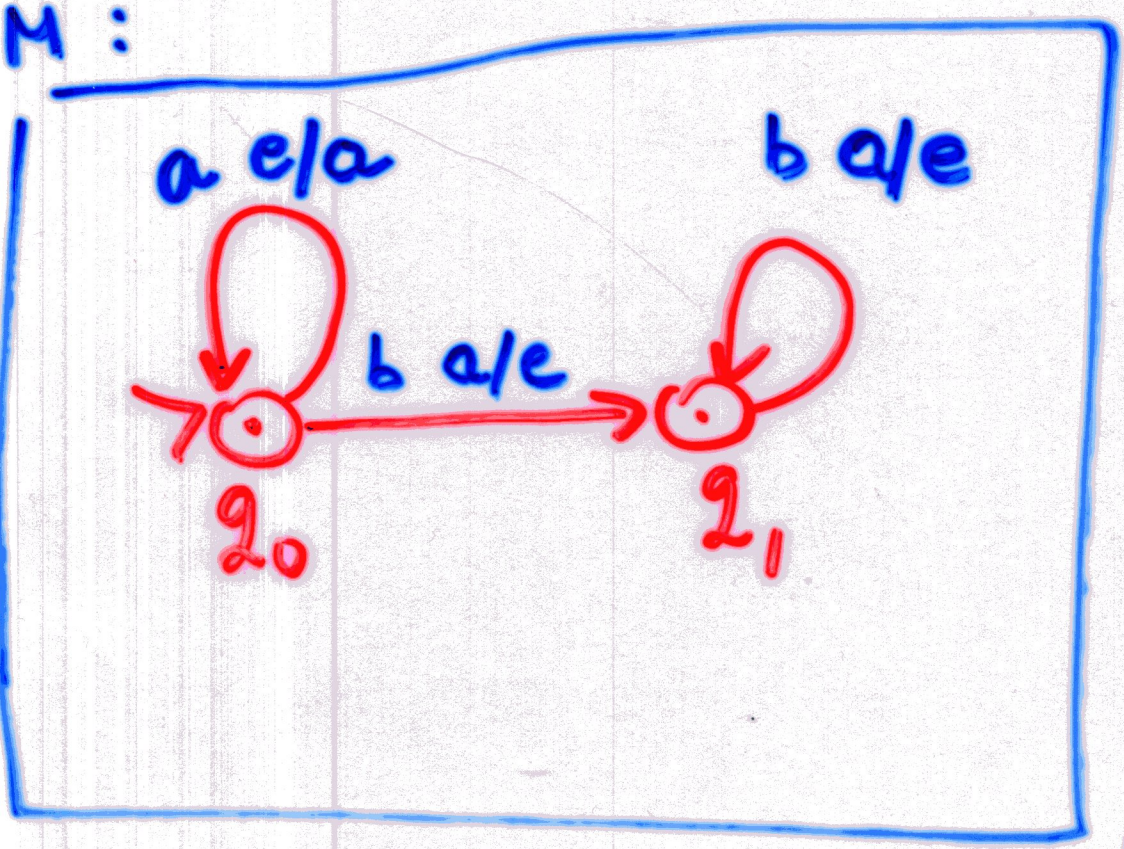
Any Finite automaton is a PD automaton

RL ⊆ CFL

EXAMPLES of Push-down AUTOMATA REVISITED

EXAMPLE 1

M:



$\Sigma = \{a, b\}$

$\Gamma = \{a\}$

$F = \{q_0, q_1\}$

$S = q_0$

Δ:

$((q_0, a, e)(q_0, a))$ PUSH a, $q_0 \to q_0$

$((q_0, b, a)(q_1, e))$ POP a, $q_0 \to q_2$

$((q_1, b, a)(q_1, e))$ POP a, $\in q_1$

$L(M) = \{a^n b^n : n \in \mathbb{N}\}$

SHOW

$aabb \in L(M)$

q_0	$aabb$	e
q_0	abb	a
q_0	bb	aa
q_1	b	a
q_1	e	e

In General

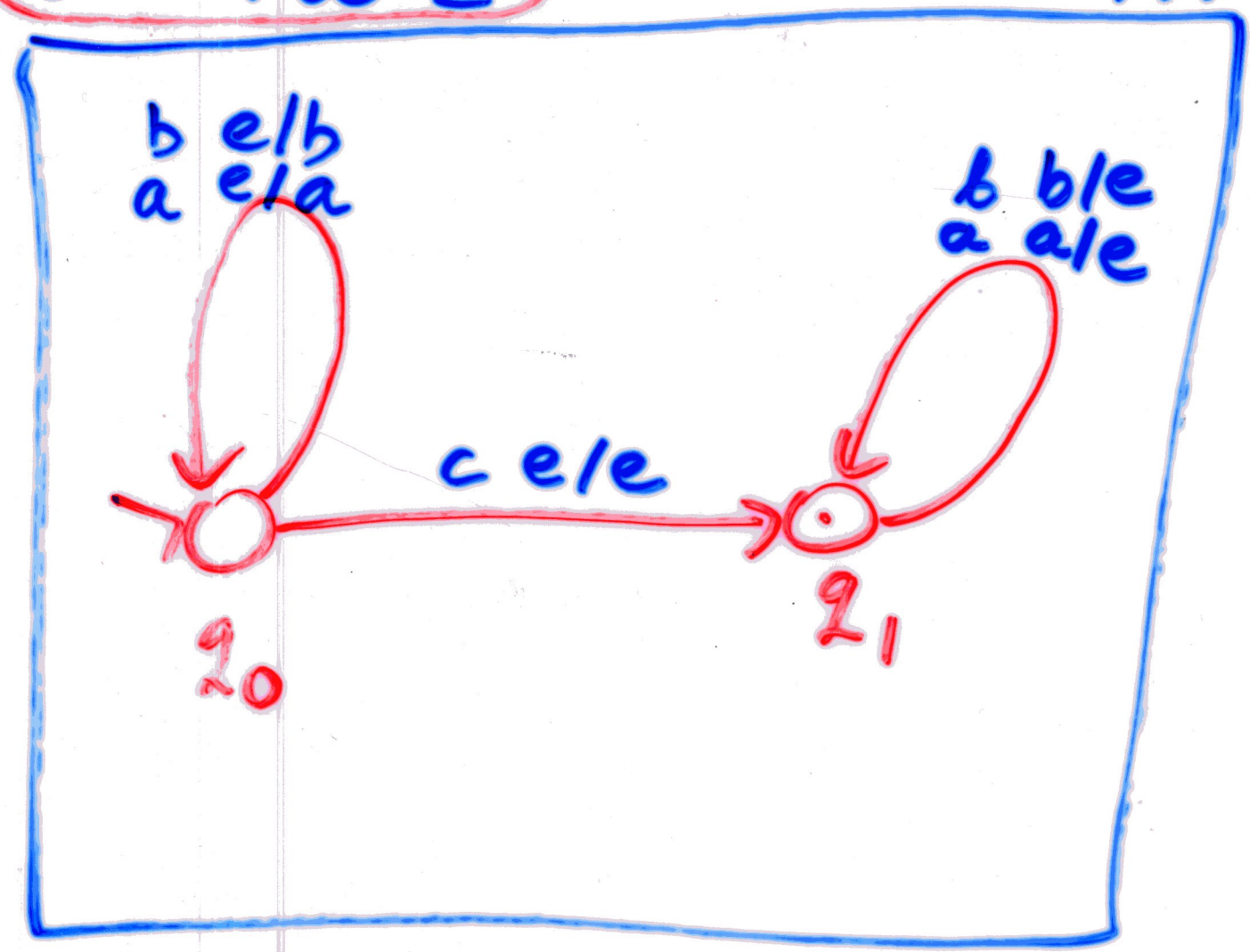
$$L(M) = \{a^n b^n : n \geq 0\}$$

$n=0$ $e \in L(M)$ because $q_0 \in F$.

Computation of length 0.

EXAMPLE 2

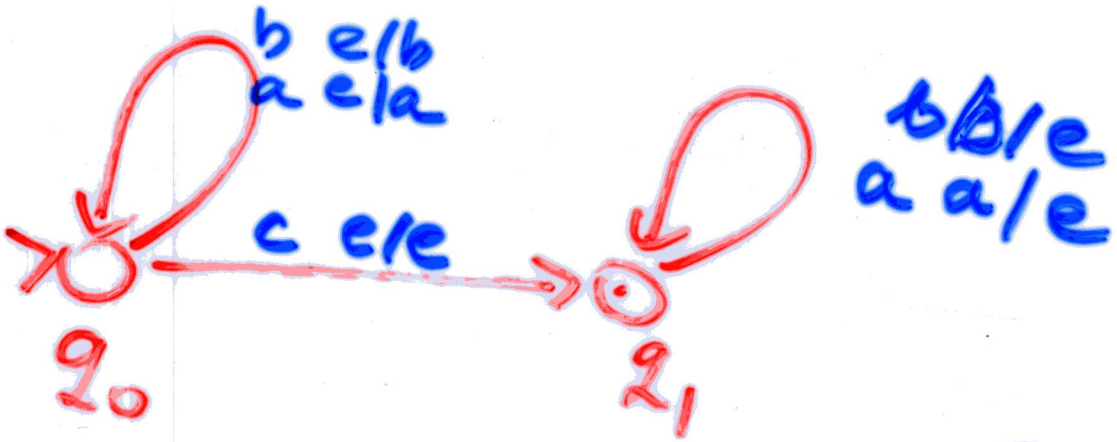
M:



- WRITE Full definition of M
- Show that

  babbacabbab $\in L(M)$

- $L(M) = \{wcw^R : w \in \{a,b\}^*\}$



$L = wcw^R$

$\Sigma = \{a, b, c\}$
$\Gamma = \{a, b\}$

Δ:

① $((q_0, a, e)(q_0, a))$ } PUSH a

② $((q_0, b, e)(q_0, b))$ } PUSH b

③ $(q_0, c, e)(q_1, e))$ go to final state when you see c

④ $((q_1, a, a)(q_1, e))$ compare a and POP a

⑤ $((q_1, b, b)(q_1, e))$ compare b and POP b

⑥ ① + ② put what we are reading from input on the stack (inverse order)

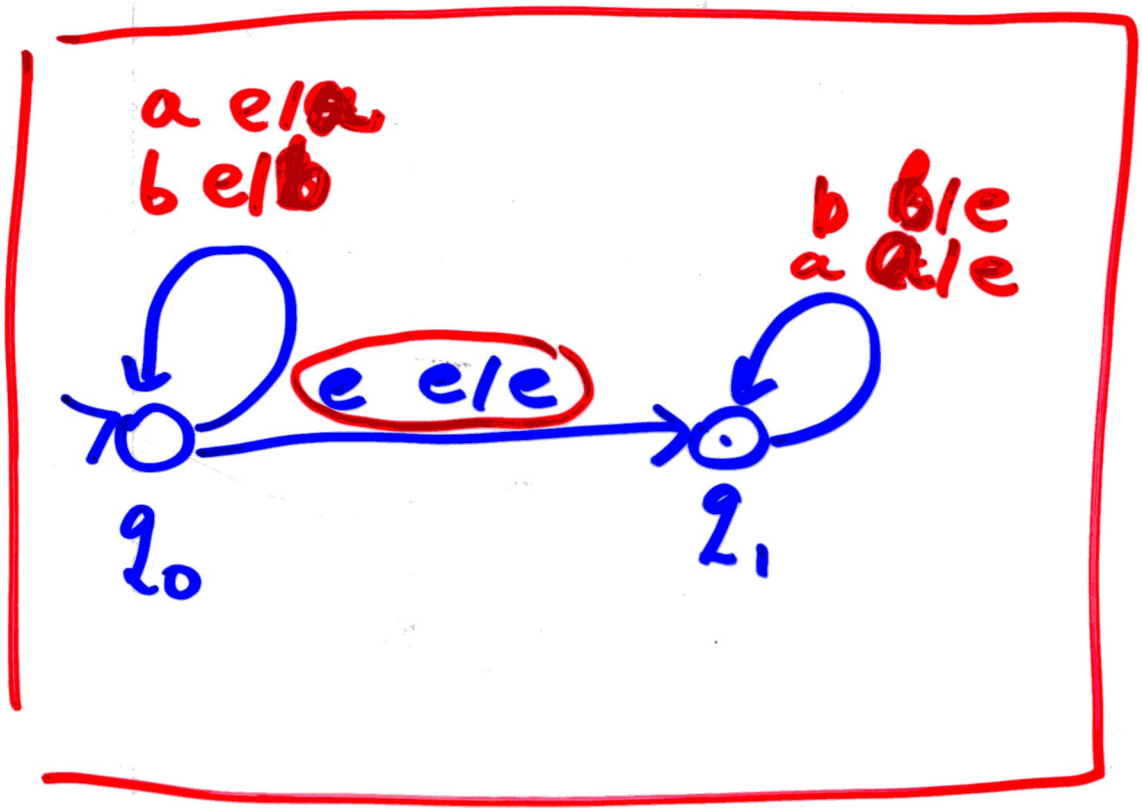
⑦ switches to FINAL

① + ③ COMPARES $w \leftrightarrow w^R$

q_0	babbacabbbab	e
q_0	abbacabbbab	b
q_0	bbacabbbab	ab
q_0	bacabbbab	bab
q_0	acabbbab	bbab
q_0	cabbbab	abbbab
q_1	abbbab	abbbab
q_1	bbbab	bbbab
q_1	bbab	bbab
q_1	bab	bab
q_1	ab	ab
q_1	b	b
q_1	e	e
a_1		

Example 3

M:



Δ:

① $((q_0, a, e)(q_0, a))$ Push a

② $((q_0, b, e)(q_0, b))$ Push b

③ $((q_0, e, e)(q_1, e))$ switch to FINAL read nothing

was $((q_0, c, e)(q_1, e))$

④ $((q_1, a, a)(q_1, e))$ compare a pop a

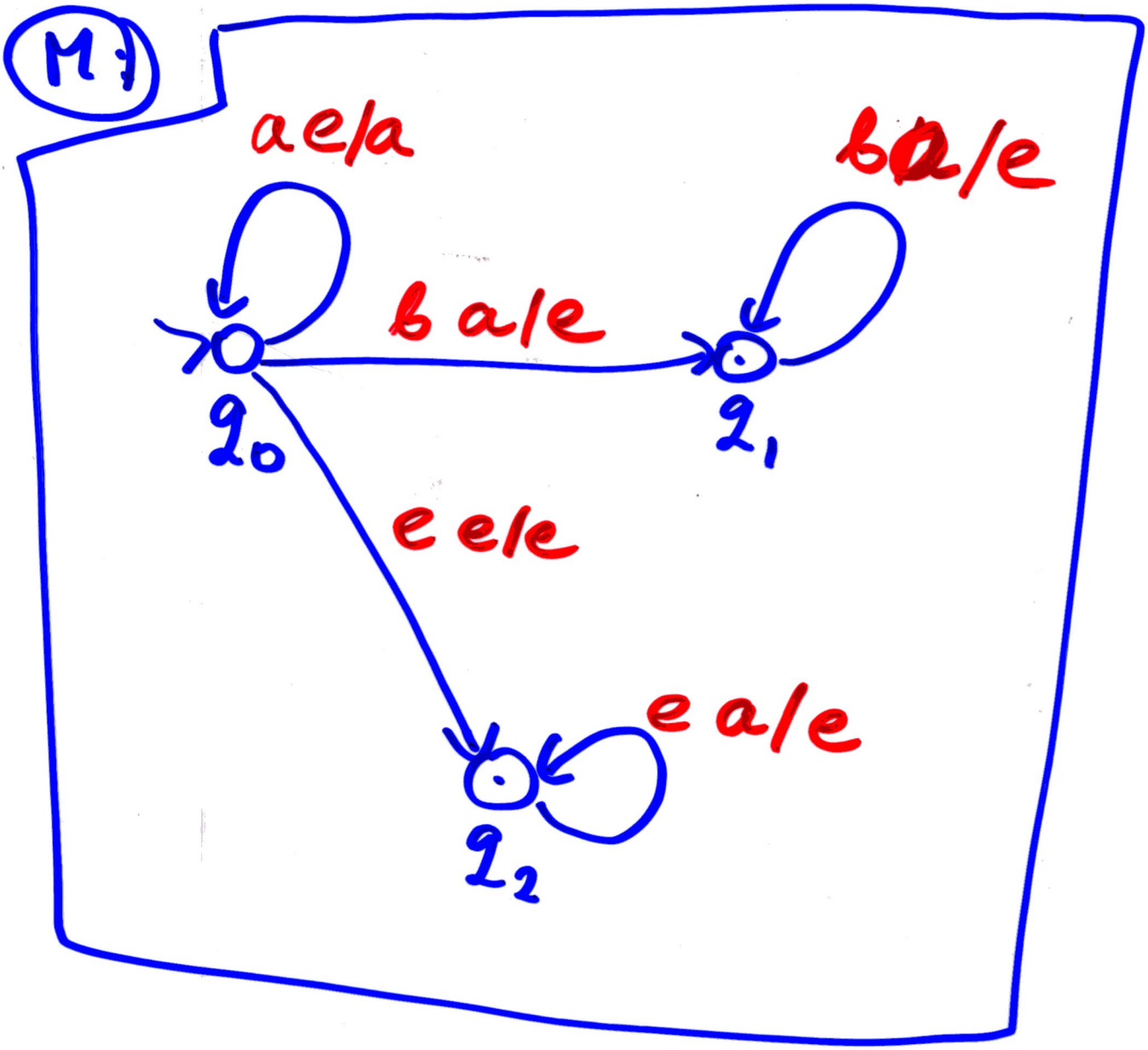
⑤ $((q_1, b, b)(q_1, e))$ compare b pop b

$$L(M) = \{ ww^R : w \in \{a, b\}^* \}$$

NONDETERMINISTIC

Example 4

NON-DETERMINISTIC



Show

$$L(M) = \{a^i : i \geq 0\} \cup \{a^i b^i : i \geq 0\}$$

$= \{w :$ w contains only a or equal # of a followed by b' s $\}$

④ $A \rightarrow aaS$

$S \rightarrow A$

$(q_1, e, A)(q_1, aaS)$

$(q_1, e, S)(q_1, A)$